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Phenomenology of $g_1(x)$ in the Observed Small- x Region

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Abstract

We examine the behaviour of the polarised structure function data for $g_1(x)$ in the region of small x (~ 0.01 to 0.1) for the various available targets (proton, neutron and deuteron) with the aim of clarifying what may be safely deduced with regard to the relation of the currently attained small- x region and the asymptotic behaviour as $x \rightarrow 0$. We find that fits using a single power-like term are susceptible to the isospin combination used. Double-power fits are more stable and also provide some evidence for an underlying SU(2) symmetry of the structure functions.

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1 Introduction

As the data on polarised nucleon deep-inelastic structure functions improves in precision, it becomes ever-more desirable to understand the control that is possible over the asymptotic behaviour as $x \rightarrow 0$ for fixed Q^2 . This desire is driven by the necessity to extrapolate to the point $x = 0$ in order to compute (or estimate) integrals of the structure functions (as required to test, *e.g.*, the Bjorken sum rule). Indeed, while it is generally held that such integrals are convergent, a steeply rising behaviour of the type $x^{-0.9}$, for example, would leave a significant part of the related integral in the unmeasured region and could invalidate any deductions derived therefrom.

The problem has already been examined [1, 2] using varying degrees of model-dependent analysis. In this letter we shall attempt to analyse the situation starting from the least model-dependent approach and then investigate the effects of various possible assumptions. In particular, we shall examine to what extent simultaneous double-power fits of all the data provide a more coherent picture as compared to combinations of single-power fits to the single data sets; and what indications there exist for an underlying isospin structure.

We shall avoid any direct reference to Regge theory (save that the structure functions should behave as powers of x asymptotically), the effect of including model input from Regge-based analyses has been discussed in [2]. We note in passing that in [2] the basic input was a logarithmic behaviour of the leading isoscalar contribution. However, given the very short lever-arm available in x (the SLAC data between about $x = 0.01$ and 0.15 were used), any logarithmic variation can easily be approximately reproduced by a power behaviour and is therefore not to be considered fundamental at this stage of the analysis.

The layout of this letter is as follows: in Section 2 we discuss the stability of single-power fits to differing combinations of the data, in Section 3 the analysis is extended to include double-power contributions and in Section 4 the effect of isospin assumptions is examined. We end with a series of conclusions and comments.

2 Single-Power Fits

In [1] an effectively single-power fit for g_1^n was performed with the conclusion that the dominant power behaviour was of the form

$$g_1^n \simeq -0.015x^{-0.87}. \quad (1)$$

If such were the case, the unmeasured region, below $x \simeq 0.01$ say, would contribute ~ 0.06 to the g_1^n integral. Since even the region below $x \simeq 0.001$ would still contribute ~ 0.05 , this would also render any future test of the such sum rules on very unsure footing. To save the Bjorken integral from unwanted implications, in [1] the authors were careful to estimate the triplet $g_1^p(x) - g_1^n(x)$ combination independently with a smaller, negative, power term ~ -0.45 .

Thus, to provide a starting point, we first perform individual fits to the SLAC E143 data at 5 GeV^2 for the three experimental targets: proton, neutron and deuteron [3] using a single-power contribution of the form

$$g_1 = \alpha x^\delta, \quad (2)$$

where α and δ are then free parameters. The results are summarised in Table 1, where we also include the hypothetical isovector target $p - n$. All fits return perfectly acceptable values of χ^2 although the powers found differ widely. A comparison of the fitted curves and the data is presented in Fig. 1

Table 1: Results for single-power fits to the SLAC E143 proton, neutron and deuteron data, at 5 GeV^2 as described in the text.

target	α	δ
g_1^p	0.18 ± 0.05	-0.24 ± 0.11
g_1^n	-0.02 ± 0.02	-0.81 ± 0.40
g_1^d	0.72 ± 1.05	0.78 ± 0.61
$g_1^p - g_1^n$	0.10 ± 0.05	-0.51 ± 0.18

As is clear from the results displayed in Table 1, the precise power returned in a single-power fit is highly dependent on the target combination used for the data, varying between -0.81 for the neutron and 0.78 for the deuteron. Thus, it is not possible, even in a first approximation to assume one universal power to describe this region of x . While this is clearly an indication of the necessity to use at least two powers, in view of the results of, *e.g.*, [1], where the neutron emerged well described by a single power (as too here), it will be instructive to study the variation of the power returned as a function of the target considered. To this aim we perform an analysis by fitting single powers to combinations of the data according to the formula

$$g_1^\theta = \cos \theta g_1^p + \sin \theta g_1^n \quad (3)$$

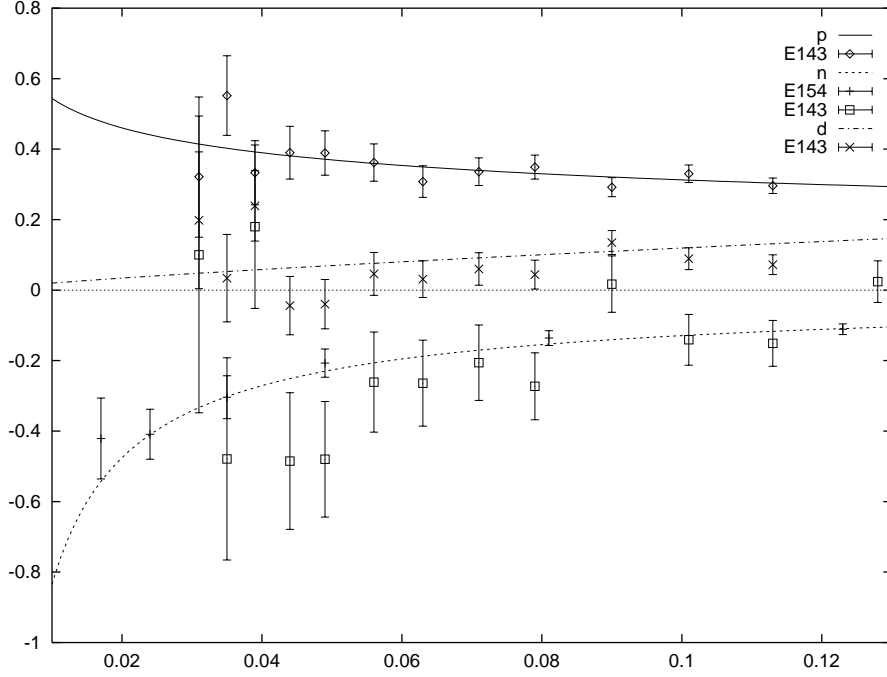


Figure 1: Comparison of the single-power fitted curves and the data.

and vary the parameter θ from 0 to 180° , thus covering the full range including both g_1^d and the combination $g_1^p - g_1^n$. The results are displayed in Fig. 2. The obvious feature is the sharp discontinuity located at $\theta \sim 120^\circ$, which clearly indicates the failure of a single-power fit in that region of combinations, very close to the deuteron. What should also not be underestimated is the tail of this discontinuity, which extends into the region of pure g_1^n .

As commented near the end of the paper, the few (2 or 3) smallest- x data points (below $x \sim 0.03$) of the E155 [4] (and SMC) experiments (at 5 GeV^2) hint that more small- x data in the future may push the deuteron fitted curves towards negative powers, which on the θ plot of a single-power fit would mean a discontinuity and thus inconsistency.

A consistent set of fits, related to the procedure of [1], is provided by taking the functions fitted to g_1^n and $g_1^p - g_1^n$ as “primary” and obtaining g_1^p and g_1^d as linear combinations, so that there are only two independent powers (all functions have acceptable χ^2). As the starting point may be any two of the four functions (with single powers), there is to be an element of arbitrariness here. Such arbitrariness is implicitly exploited in [1] where the single-power fit to g_1^n is considered more fundamental or “primary”, thus leading to a very divergent g_1^n . However, such behaviour disappears if g_1^n is

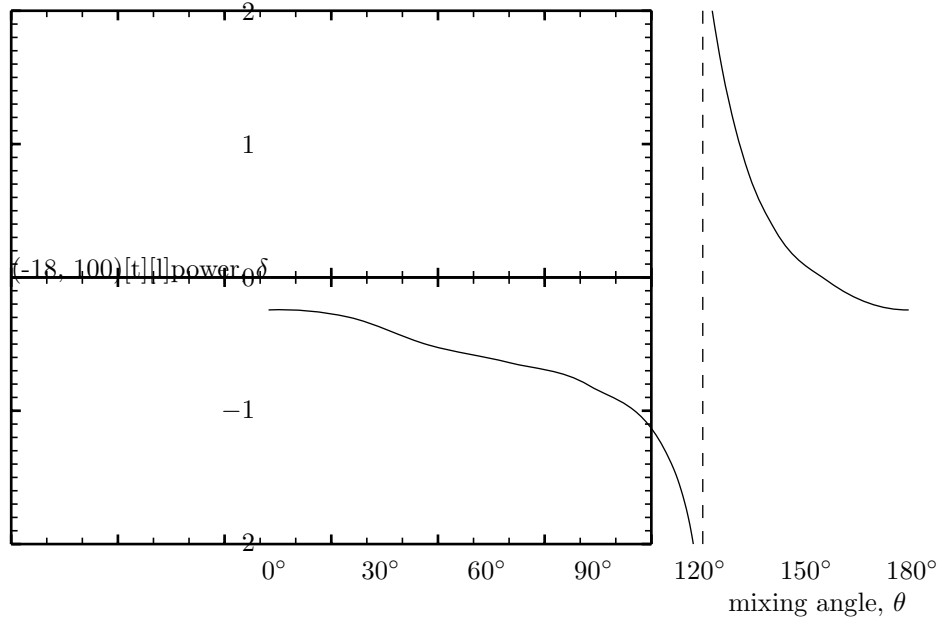


Figure 2: The power, δ , obtained in a single-power fit as a function of θ , the mixing parameter for g_1^p and g_1^n described in the text.

taken as a linear combination, as also intuitively suggested in [5]. Is there anything then that favours two of the functions and their associated single-powers as more fundamental than the other two?

Here, we are led once again to consider two separate power-term contributions, at least in two of the four combination g_1 functions considered. Thus, it is reasonable to begin with a two-power global fit to the structure functions to avoid all arbitrariness. This ought to be done simultaneously for all the available data. We actually used SLAC and HERMES at approximately 3 GeV^2 .

Before moving on to double-power fits, it is worth investigating the possibility of describing both the proton and neutron data with the same single power (*i.e.*, explicitly leaving aside the deuteron data for the moment); the results are

$$\begin{aligned} \alpha_p &= 0.10 \pm 0.02 \\ \alpha_n &= -0.05 \pm 0.01 \\ \delta &= -0.48 \pm 0.08. \end{aligned} \tag{4}$$

However, the χ^2 here is significantly poorer than in any other fit so far. On examining these results together with the data, it emerges that the poor fit could be readily remedied by the inclusion of a plateau increasing the overall proton polarisation and decreasing that of the neutron. The effect would then

be to reduce the neutron power while increasing that of the proton, leaving a slightly positive deuteron (given by the plateau contribution itself).

3 Double-Power Fits

Not only is it obviously necessary to allow for more than one power in the fits, but it should also be clear that any single power returned above could change dramatically in a two-power fit [5]. In particular, two facts lead to the possibility that any given behaviour might be mimicked by combinations of different powers.

First of all is the limited lever-arm available in terms of the x -range of the data one can sensibly use; little more than one order of magnitude variation in x is the bare minimum for extracting a power behaviour, given the present experimental errors. Second, and often neglected, is the fact that, in contrast to the unpolarised case, where most of the standard wisdom has been gathered, the polarised structure functions are *not* constrained to be positive definite. This means that, *e.g.*, an overall rising behaviour (in magnitude) may be due either to a single rising contribution *or* to a rising cancellation between two relatively changing contributions of opposite sign, neither of which need necessarily be similarly rising.

In [5] an explicit example was invented *ad hoc*, in which a behaviour of the type (as proposed in [1])

$$g_1^n(x) \sim \frac{-0.02}{x^{0.8}} \quad (5)$$

was shown to be well reproduced, within experimental errors and over the finite range of x considered, with the form

$$g_1^n(x) \sim \frac{-0.07}{x^{0.5}}(1 - 4x), \quad (6)$$

which, one should note, has very different asymptotic behaviour.

The fact that single-power fits do, in fact, work rather well implies that an attempt to fit any single target data set (excepting perhaps that of the deuteron) with two power terms will encounter serious difficulties. Indeed, the only way to perform successful two-power fits is to combine data sets of different targets and demand that the two powers used be the same. The results of such a fit can be summarised as follows: for the form

$$g_1 = \alpha x^\delta + \beta x^\gamma, \quad (7)$$

and the SLAC [3, 6] E143, E154 together with the HERMES [7] data sets, we obtain

$$\begin{aligned}
\alpha_p &= 0.01 \pm 0.02 \\
\alpha_n &= -0.02 \pm 0.03 \\
\delta &= -0.77 \pm 0.33 \\
\beta_p &= 0.26 \pm 0.20 \\
\beta_n &= 0.00 \pm 0.10 \\
\gamma &= 0.13 \pm 0.50
\end{aligned} \tag{8}$$

and again the χ^2 is perfectly acceptable. Note that the large errors are mainly due to strong correlations between coefficients and powers and therefore ultimately between the coefficients themselves. Again, a comparison of the fits and data is shown in Fig. 3.

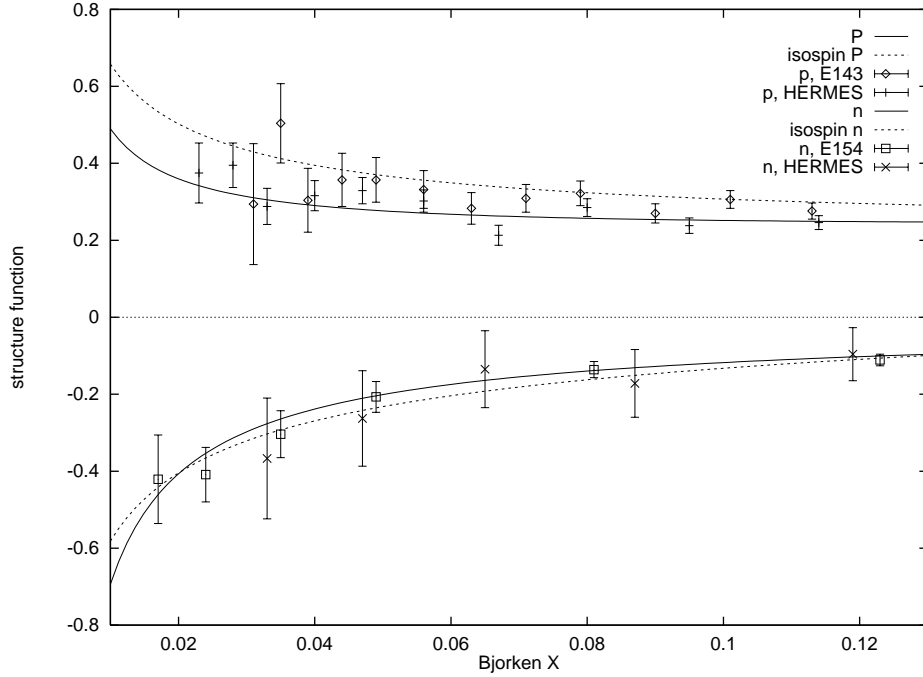


Figure 3: Comparison of the double-power fitted curves (solid lines) and isospin type fits (dashed lines) with the data.

One sees the attraction of the neutron data to a single-power fit, *i.e.*, that a second power is not at all required by this particular target. Moreover, the fact that the power chosen is very close to the original power but far from that of the proton is indicative of the possibility to mimic powers (in this case that of the proton) by combining different contributions. Note also that, as has been found by various authors, the steeper behaviour (δ here), having

opposite sign coefficients (α) in the proton and neutron, is to be ascribed to an isovector contribution—within errors the two coefficients have the same magnitude.

As a final test we fix the steep power to be less divergent, setting $\delta = -0.5$, and repeat the fits—the idea being to present a picture less clouded by the correlations between parameters, which often artificially inflate errors; the results are

$$\begin{aligned}
\alpha_p &= 0.03 \pm 0.05 \\
\alpha_n &= -0.07 \pm 0.03 \\
\delta &= -0.50 \quad \text{fixed} \\
\beta_p &= 0.17 \pm 0.09 \\
\beta_n &= 0.11 \pm 0.07 \\
\gamma &= 0.02 \pm 0.63
\end{aligned} \tag{9}$$

as always the χ^2 is perfectly acceptable and indeed marginally improves due to the reduced number of parameters.

Hence it is clear that such steep powers as previously found are not an absolute requirement of the data, as suggested by the large errors found earlier. Moreover, on choosing a less divergent behaviour the various coefficients come more into line with isospin symmetry requirements; *i.e.*, one term is approximately isovector and the other isoscalar.

4 Isospin Symmetric Fits

In the light of the above fits, it is natural to enquire as to the effect of requiring that the two terms used fall precisely into the two categories of isovector and isoscalar. Thus, as a final test we have simply repeated the above fit fixing $\alpha_n = -\alpha_p$ and $\beta_n = \beta_p$. The results are (with the usual good χ^2)

$$\begin{aligned}
\alpha_p &= 0.78 \pm 0.02 \\
\delta &= -0.45 \pm 0.07 \\
\beta_p &= 0.20 \pm 0.14 \\
\gamma &= 0.36 \pm 0.28,
\end{aligned} \tag{10}$$

where δ and γ now refer to the isovector and isoscalar powers respectively.

Relaxing one or other of the isospin constraints leads to similar fits with similar χ^2 . Note that in this case the steep power, which is, of course, now constrained to be that of the combination $g_1^p - g_1^n$ is much less steep than in many other fits. This would mean an increasing difference in total polarisation of u and d quarks, with decreasing x in the given interval. A comparison of the fits and data can be found in Fig. 3.

Thus, we may say that in the small- x region under examination, the double-power fits of the data prior to the E155 experiments are consistent with the assumption that the positive isotriplet contribution is causing the mildly divergent behaviour of $g_1^p(x)$ and $g_1^n(x)$. However, the latest E155 data on the deuteron [4], which are still preliminary, provide a hint of a negative singlet term in $g_1^d(x)$ with a more strongly divergent behaviour, setting in below $x = 0.03$, in accordance with the expected [6] asymptotic dominance of the singlet. This, together with the observed attraction of the neutron data to a single steep power, calls for a fresh look at the picture when the final E155 data become available.

5 Conclusions

The first lesson that we wish to underline is the possibility that any result arising from an effectively *single*-power fit may be biased precisely by the choice of a single term to describe the data. Thus, although the neutron data appears to select a large single power, two lesser powers are also perfectly acceptable (and less dramatic). Indeed, by comparing the magnitudes of the isoscalar and isovector contributions, one sees that over much of the x range of interest neither contribution is negligible. It is also clear that the presence of an isoscalar plateau leads to a steepening of one power and a flattening of the other.

Perhaps, the second lesson is that, while the isospin properties of the distributions are still far from well defined in the data, a simple description based on such a symmetry does, in fact, work rather well.

One caveat that should be borne in mind is that we have not considered how evolution might alter the picture: this is both beyond the scope of the present letter and, to any reliable degree, beyond the level of the present data given the poor determination of the gluon distribution.

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